Important points in tennis

It is possible to calculate the probability of a player winning a game from any particular score. We denote the probability of the server winning each point as p, where $0 \leq p \leq 1$. For example, a value of $p = 0.9$ indicates that the server is 90% likely to win any given point.

Table 1 lists the probabilities of the server winning the game from each possible score, for three example values of p.

Case I: $p = 0.75$

In Case I, the server wins three-quarters of all points on average. This usually corresponds to the performance of a strong server in a decisive straight-sets victory. Table 1 shows that the server is broken in only 5% of service games. Interestingly, it also shows that the players are equally likely to win the game when the score is $15 - 40$.

Case II: $p = 0.62$

This case corresponds to a 'typical' match. The value $p = 0.62$ was the average probability across the entire ATP circuit in 2010, and it often arises when the players are evenly matched.

Case III: $p = 0.55$

In this case the server is extremely weak, winning only 55% of points on serve. Consequently, breaks of serve occur in 38% of service games. This scenario often arises in straight-sets defeats, where the receiving player is vastly superior.

Table 2 gives the general expressions for the probability of the server winning the game from each possible score.

Score	Probability of server winning game		
	$p = 0.75$	$p = 0.62$	$p = 0.55$
$0 - 0$	0.949	0.776	0.623
$0 - 15$	0.870	0.623	0.458
$0 - 30$	0.696	0.411	0.271
$0 - 40$	0.380	0.173	0.100
$15 - 0$	0.976	0.870	0.758
$15 - 15$	0.928	0.752	0.611
$15 - 30$	0.802	0.557	0.411
$15 - 40$	0.506	0.279	0.181
$30 - 0$	0.991	0.942	0.879
$30 - 15$	0.97	0.872	0.775
$\begin{cases} 30 - 30 \\ 40 - 40 \end{cases}$	0.900	0.727	0.599
$\begin{cases} 30 - 40 \\ 40 - \text{Ad} \end{cases}$	0.675	0.451	0.329
$40 - 0$	0.998	0.985	0.963
$40 - 15$	0.994	0.961	0.919
$\begin{cases} 40 - 30 \\ \text{Ad} - 40 \end{cases}$	0.975	0.896	0.820

Table 1: Probability of the server winning the game from each score, for the three example values of p.

Score	Probability of server winning game
$0-0$	$\dfrac{p^4\,(15-34\,p+28\,p^2-8\,p^3)}{2\,p^2-2\,p+1}$
$0-15$	$\dfrac{p^4\,(5-4\,p-2\,p^2+2\,p^3)}{2\,p^2-2\,p+1}$
$0-30$	$\dfrac{p^4\,(1+2\,p-2\,p^2)}{2\,p^2-2\,p+1}$
$0-40$	$\dfrac{p^5}{2\,p^2-2\,p+1}$
$15-0$	$\dfrac{p^3\,(10-25\,p+26\,p^2-12\,p^3+2\,p^4)}{2\,p^2-2\,p+1}$
$15-15$	$\dfrac{p^3\,(2\,p^2-5\,p+4)}{2\,p^2-2\,p+1}$
$15-30$	$\dfrac{p^3\,(1+p-p^2)}{2\,p^2-2\,p+1}$
$15-40$	$\dfrac{p^4}{2\,p^2-2\,p+1}$
$30-0$	$\dfrac{p^2\,(6-16\,p+19\,p^2-10\,p^3+2\,p^4)}{2\,p^2-2\,p+1}$
$30-15$	$\dfrac{p^2\,(3-5\,p+4\,p^2-p^3)}{2\,p^2-2\,p+1}$
$\begin{cases}30-30\\40-40\end{cases}$	$\dfrac{p^2}{2\,p^2-2\,p+1}$
$\begin{cases}30-40\\40-\text{Ad}\end{cases}$	$\dfrac{p^3}{2\,p^2-2\,p+1}$
$40-0$	$\dfrac{p\,(3-8\,p+10\,p^2-5\,p^3+p^4)}{2\,p^2-2\,p+1}$
$40-15$	$\dfrac{p\,(2-4\,p+4\,p^2-p^3)}{2\,p^2-2\,p+1}$
$\begin{cases}40-30\\\text{Ad}-40\end{cases}$	$\dfrac{p\,(1-p+p^2)}{2\,p^2-2\,p+1}$

Table 2: Probability of the server winning the game from each score, for a given probability p of the server winning each point.

So, which are the most important points? It varies by the strength of the server, as shown in Figure 1. When the server is strong, you should still pay attention if the score is $0 - 30$ or $0 - 40$. When the server is weak, the $15 - 30$, $30 - 30$, $30 - 40$ and $40 - 30$ points are more important.

Score	p = 0.75	p = 0.62	p = 0.55
00-00	Low	Medium	Medium
00-15	Medium	Medium	Medium
00-30	High	Medium	Medium
00-40	High	Medium	Low
15-00	Low	Low	Medium
15-15	Low	Medium	Medium
15-30	Medium	High	High
15-40	High	High	Medium
30-00	Low	Low	Low
30-15	Low	Medium	Medium
30-30	Medium	High	High
30-40	High	High	High
40-00	Low	Low	Low
40-15	Low	Low	Low
40-30	Low	Medium	High

Figure 1: Importance of each point in the game, given the strength of the server. This is calculated by as the difference of probabilities of each player winning the game if they win this point.