Optimal serving at the Australian Open

1 Background

The tactic of using a strong first serve and a weaker, but more reliable, second serve is well established in tennis. But is it really the best tactic? Could two strong serves, or two weak serves, or some other combination be more successful?

The theory of service strategy (see Section 2) is simple, and was summarised in a two-page paper by Gale in 1971. Since then it has been reproduced by about six different authors who must have thought that using different pronumerals counted as novel research. But nobody has ever applied the theory to actual match data, so I'm interested in seeing how it stacks up.

2 Theory

First, some definitions:

- Let s_1 be the probability of a player's first serve landing in.
- Let w_1 be the probability of a player winning the point on their first serve, given that the serve landed in.
- Let s_2 be the probability of a player's second serve landing in.
- Let w_2 be the probability of a player winning the point on their second serve, given that the serve landed in.

The parameters s_1 and s_2 can be considered constant for a particular player, whereas w_1 and w_2 depend on the skill of the receiver. The probability of a player winning a point on their serve is then

$$p = s_1 w_1 + (1 - s_1) s_2 w_2. \tag{1}$$

To find the strategy that maximises p, we begin by considering the second serve. The probabilities of the first serve are obviously irrelevant here, so p is maximised when $s_2 w_2$ is a maximum — that is, when the total proportion of points won on the second serve is a maximum. This makes perfect sense.

What about the first serve? Letting α^* be the maximum value of $s_2 w_2$, then the maximum value of p in (1) is given by

$$\max p = \max s_1 (w_1 - \alpha^*). \tag{2}$$

Figure 1 shows the implications of (2) on the service strategy. Here we assume that a player possesses a continuum of serves for which the success rate w is a decreasing function $f(s)$ of the service probability. The graph demonstrates that the optimal first serve is riskier (but more successful) than the optimal second serve. Furthermore, the greater the value of α^*, the riskier the optimal first serve is.

This shows that it is actually the winning percentage of the *second* serve that determines how risky the first serve can afford to be.

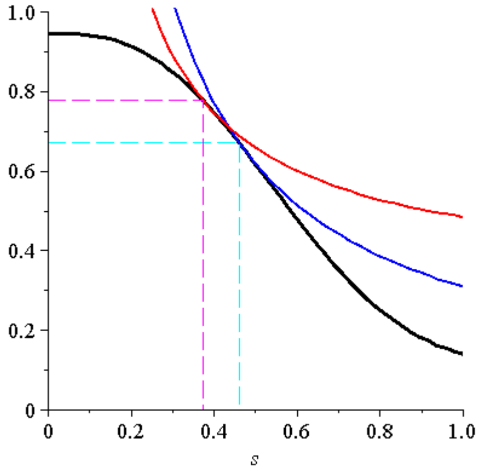

Figure 1: Optimal first-serve strategy. The black line represents $w = f(s)$ for a player. The dark blue curve is the maximum of sw along $f(s)$, giving an optimal second serve of $(s_2^*, w_2^*) \approx (0.46, 0.67)$. The red curve is the maximum of $s(w - \alpha^*)$ along $f(s)$, and yields and optimal first serve of $(s_1^*, w_1^*) \approx (0.37, 0.77)$.

Because players only really have two serves in their armoury, we only know two points on the curve $w = f(s)$. It follows from (1) that a player is better off using two strong serves (that is, $s_2 := s_1$ and $w_2 := w_1$) if

$$s_1 w_1 > s_2 w_2, \tag{3}$$

which obviously means that the weak serve does not optimise sw.

Conversely, a player should employ two weak serves if

$$s_1 w_1 < s_2 w_2 (1 - (s_2 - s_1)). \tag{4}$$

Finally, we note that the probability of a player winning a game on serve is

$$g = \frac{p^4 \left(15 - 34\,p + 28\,p^2 - 8\,p^3\right)}{2\,p^2 - 2\,p + 1}, \tag{5}$$

where p is the probability of winning a point on serve, as defined in (1).

3 Australian Open 2012

3.1 Summary

The official service statistics were collected for every men's and women's singles match at the Australian Open 2012. Three service strategies are considered: *SS* (two strong serves), *SW* (the traditional strong serve followed by a weak serve) and *WW* (two weak serves).

Table 1 shows the average probability of winning a service game using each strategy across the entire tournament, calculated on the assumption that every player used a strong serve followed by a weak one. SW is clearly the best strategy, for the reasons explained in the previous section, followed by SS. Two weak serves is a weak strategy, winning little more than half of all games on average. The relative differences between strategies is very similar for men and women.

	SS	**SW**	**WW**
Men	70.9%	76.4%	58.9%
Women	56.7%	62.2%	50.2%
Overall	65.8%	71.4%	55.8%

Table 1: Average probability of holding serve using each strategy.

3.2 Service strategy by result

But that isn't to say that SW is always the best strategy. We can use the inequalities (3) and (4) to determine the optimal strategy for each match, given a player's service data. Table 2 lists the proportion of matches in which each strategy is optimal. The SW approach is optimal in 68% of men's matches but only 41% of women's matches, so it is more likely to work for men.

These results also show that SW is an especially dominant strategy for players who win. The other tactics are more likely to be optimal if a player is on the losing end of the scorecard. In particular, the distribution of optimal strategies for losing female players is $(SS, SW, WW) = (34\%, 36\%, 30\%)$. This indicates that there is no general dominant approach for women getting beaten — in many cases, they might as well toss a coin before each serve to decide which serve to send down. It would make little difference to their chance of winning the game.

Result	**Men**	**Women**	**Overall**
Win	$(20, 74, 6)$	$(24, 46, 32)$	$(21, 60, 19)$
Loss	$(27, 62, 11)$	$(34, 36, 30)$	$(30, 49, 21)$
Overall	$(23, 68, 9)$	$(29, 41, 30)$	$(26, 54, 20)$

Table 2: Optimal service strategy by draw and result. Ordered triplets are given as (SS, SW, WW) where the values represent the percentage of matches in which each service strategy is optimal.

Breaking this analysis down further, Table 3 examines the effect of the match margin. It is hard to identify a trend here. The SW tactic is optimal for 82% of men who win narrowly (four or five sets), but only 39% of women who win in a deciding set. We can again see that SW is more likely to be the best approach for players who win, and it is also more important for men to use this strategy.

There were no significant patterns when optimal service strategy was analysed by the tournament round.

Result	Men	Women
Straight-set win	$(23, 69, 8)$	$(19, 47, 34)$
Narrow win	$(15, 82, 3)$	$(39, 39, 22)$
Narrow loss	$(27, 68, 5)$	$(36, 36, 28)$
Straight-set loss	$(28, 56, 16)$	$(34, 37, 29)$

Table 3: Optimal service strategy by draw and result margin. Ordered triplets are given as (SS, SW, WW) where the values represent the percentage of matches in which each service strategy is optimal.

3.3 Service strategy by player

Finally, we consider the service strategies of individual players throughout the tournament. Table 4 lists the five players who would have gained the most from using strong serves on both their first and second attempts. Nicolas Mahut, who famously lost a 11-hour match at Wimbledon 2010, is an interesting inclusion. Across three matches and almost 300 serves, he would have been 11.4% more likely to hold serve had he opted for the SS approach. This reflects the power and success rate of his first serve compared to his second. It also explains why it took John Isner 69 games to break him in the fifth set at Wimbledon!

However, further analysis of Mahut's recent record shows that SS is not always his optimal strategy. Of his last 45 ATP matches, the traditional SW was his best ploy on 27 occasions, compared to 10 matches in which SS would have been better, and a surprising 8 matches in which two weak serves would have improved his chances.

Draw	Player	Matches	Serves	g_{SW}	g_{SS}	Improvement
M	Mannarino	1	101	70.2%	78.7%	12.1%
M	Mahut	3	298	60.8%	67.7%	11.4%
W	Hantuchova	3	270	54.6%	60.8%	11.4%
W	Peng	2	147	67.3%	73.3%	8.8%
W	Cibulkova	2	152	71.9%	77.8%	8.2%

Table 4: Players who would have realised the greatest improvement from using an SS strategy at the 2012 Australian Open (minimum 100 points on serve), where g_{SW} and g_{SS} are the probabilities of winning games using SW and SS strategies respectively.

On the other hand, Table 5 shows players who would have benefited most from using two weak serves. The list is dominated by poorly-performed players. The magnitudes of the improvement percentages are much higher than in Table 3, since a poor first serve is more harmful to a player's chances than a poor second serve.

Draw	**Player**	**Matches**	**Serves**	g_{SW}	g_{WW}	**Improvement**
W	Cornet	1	109	37.6%	51.6%	37.2%
M	Cipolla	2	234	61.6%	80.9%	31.4%
M	Prodon	1	127	71.6%	92.8%	29.7%
W	Dubois	2	179	45.1%	55.1%	22.2%
W	Tatishvili	2	140	68.9%	84.0%	21.8%

Table 5: Players who would have realised the greatest improvement from using a WW strategy at the 2012 Australian Open (minimum 100 points on serve), where g_{SW} and g_{WW} are the probabilities of winning games using SW and WW strategies respectively.